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Probability of error for LDPC –OC with one co-channel Interferer over i.i.d Rayleigh Fading

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Abstract: *An analytical approach is used to derive bit error rate for LDPC coded-optimum combined (LDPC-OC) system in the presence of only single co-channel interferer. A probability density function has also been derived using moment generating function for optimum combining with one interferer. This paper considered that signal is BPSK modulated and transmitted over i.i.d Rayleigh fading channel. The analytical result shows that for BER 10^{-2}, the LDPC-OC system provides an additional gain of 4.1 dB over OC system alone.*
Keywords: *irregular low-density parity check (LDPC) codes, optimum combining, rayleigh fading channel*

I. Introduction

The optimum combining is popular diversity combining technique while working in the interference scenarios. As compared to MRC system, OC has achieved a large output SINR thereby providing a powerful means to enhance signal detection in the presence of co-channel interference [1]. On the other hand performance of LDPC codes has approached the Shannon limit on the additive white Gaussian noise (AWGN) channel [2].

In the fading environment, an exact analysis of performance of channel coded and diversity combined system is usually quite complicated and computer simulation is often used to study system performance. Computer simulation has derived the threshold SNR of regular/irregular LDPC codes for SIMO system [3]. For LDPC coded-optimum combining (LDPC-OC) system, only simulation work has been done by Surbhi Sharma [4]. The analytical efficient BER expression for LDPC coded and SC/MRC combined system has been derived [5]. The bit error rate of LDPC-SC, LDPC-MRC system is derived for BPSK modulation over an independent and identically distributed (i.i.d) Rayleigh fading channel by varying the number of receiving antenna elements from 3 to 6.[6].The analytical probability of error for LDPC-OC has been derived when number of interferers are greater than or equal to number of receiving antennas [7]. Using moment generating function approach, Wu, Yongpeng et al.[8] derived BER from SINR at the combiner output for multiple arbitrary-power interferers. In this paper probability density function has been derived using moment generating function for optimum combining and exact BER expression for LDPC coded optimally combined system (LDPC-OC) in the presence of single interferer over Rayleigh fading channel has also been derived.

II. System Model

The system model which is considered in this paper is as shown in Fig.1. In this model, irregular LDPC coded signal is transmitted and at the receiver side, M-element antenna arrays receives LDPC coded signal which is operated in the presence of one co-channel interferer.

Suppose an array of 'M' antennas is used for reception of desired signal corrupted by AWGN and single co-channel interferer. The optimum combiner combines the received signal vector after weighting which mitigate the effect of interference. The received signal at the output of combiner which is fed to the LDPC decoder is given as

$$y(t) = a^*s(t) + n(t)$$

where 'a' is the optimum combining channel gain over i.i.d rayleigh fading. s(t) is transmitted BPSK modulated signal and n(t) is additive white Gaussian noise with mean zero and variance σ_n^2.Then this received vector further decoded by LDPC decoder which based on the message passing algorithm. General idea of such a decoding algorithm is to pass messages in a cycle. Each cycle has two phases. In the first phase messages are passed from variable node 'v' of degree 'j' to check node 'c' of degree 'i' of the factor graph. In the second phase, messages are passed back from the check node 'c' to the variable node 'v'.

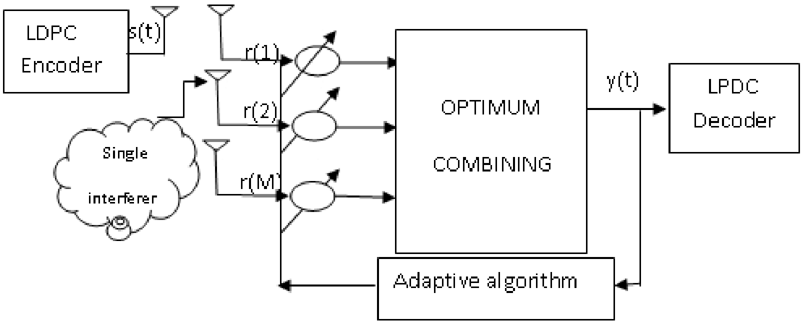

Fig.1 system model of ldpc-oc with single interferer

III. Pdf Of Uncoded Optimum Combining By Using Mgf

The moment generating function (MGF) of γ_{oc} at given γ_1 and Probability density function for SNR of interference [9, equation (10 & 8) respectively]

$$\psi_{oc}(t|\,\gamma_1) = \left(\frac{\frac{\gamma_1+1}{\bar{\gamma}_s}}{\frac{\gamma_1+1}{\bar{\gamma}_s} - t}\right)\left(\frac{\frac{1}{\bar{\gamma}_s}}{\frac{1}{\bar{\gamma}_s} - t}\right)^{M-1} \qquad : (1)$$

$$p(\gamma_1) = \frac{1}{\bar{\gamma}_1{}^{M}\,\Gamma(M)}\gamma_1{}^{M} \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1}\right) \qquad \gamma_1 \geq 0 \;; \qquad : (2)$$

Calculate the conditioned PDF of OC over γ_1 from moment generating function from equation (1) is

$$p(\gamma_{oc}|\gamma_1) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \psi_{OC}(t|\gamma_1) e^{t\gamma_{oc}}\, dt \;; \qquad : (3)$$

Put the value of equation (1) into equation (3) and using the integral [10, equation 3.384(76)], then the PDF conditioned on γ_1 is calculated as

$$p(\gamma_{oc}|\gamma_1) = \frac{(1+\gamma_1)\gamma_{oc}{}^{(M-1)}}{\bar{\gamma}_s{}^{M}} e^{\left(\frac{-\gamma_{oc}}{\bar{\gamma}_s}\right)}\, {}_1F_1\left[1; M; -\frac{\gamma_1\,\gamma_{oc}}{\bar{\gamma}_s}\right] \qquad : (4)$$

Now to get unconditioned PDF over γ_1, then PDF of SINR at the optimum combining is

$$p(\gamma_{oc}) = \int_0^{\infty} p(\gamma_{oc}|\gamma_1)\; p(\gamma_1)\, d\gamma_1 \qquad : (5)$$

Put the values of equation (2 & 4) into equation (5), then the PDF of OC becomes

$$p(\gamma_{oc}) = \frac{K_{M-1}(\bar{\gamma}_s, \gamma_{oc})}{\left(1 + \frac{\bar{\gamma}_1}{\bar{\gamma}_s}\gamma_{oc}\right)}\left[1 + M\bar{\gamma}_1 \frac{1 + \frac{(M-1)}{M}\frac{\bar{\gamma}_1}{\bar{\gamma}_s}\gamma_{oc}}{1 + \frac{\bar{\gamma}_1}{\bar{\gamma}_s}\gamma_{oc}}\right]$$

For M>>1 then above equation becomes

$$p(\gamma_{oc}) = \frac{K_{M-1}(\bar{\gamma}_s, \gamma_{oc})}{\left(1 + \frac{\bar{\gamma}_1}{\bar{\gamma}_s}\gamma_{oc}\right)}(1 + M\bar{\gamma}_1) \qquad : (6)$$

BER for uncoded OC system over the Rayleigh faded channel is

$$P_e = \int_0^{\infty} \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma_{oc}})\, p(\gamma_{oc})\, d\gamma_{oc} \qquad : (7)$$

Put the value of equation (6) into equation (7) then probability of error is calculated as

$$P_e = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_s}{\bar{\gamma}_s+1}}\sum_{k=o}^{M-2}\binom{2k}{k}\left(\frac{1}{4(\bar{\gamma}_s+1)}\right)^{k}\right] - \frac{1}{2\Gamma(M)(-\bar{\gamma}_1)^{M-1}} *$$
$$\left\{\sqrt{\frac{\pi\bar{\gamma}_s}{\bar{\gamma}_1}}\exp\left(\frac{\bar{\gamma}_s+1}{\bar{\gamma}_1}\right)\operatorname{erfc}\left(\sqrt{\frac{\bar{\gamma}_s+1}{\bar{\gamma}_1}}\right) - \sqrt{\frac{\bar{\gamma}_s}{\bar{\gamma}_s+1}}\sum_{k=0}^{M-2}\frac{(2k)!}{k!}\left(\frac{-\bar{\gamma}_1}{4(\bar{\gamma}_s+1)}\right)^{k}\right\} \qquad : (8)$$

IV. Ber For Optimum Combining With Ldpc

The conditional PDF of channel LLR over the channel gain is given by [7, equation (3)]

$$p_0(q|a, s(t) = +1) = \frac{\sigma_n}{2a\sqrt{2\pi}}\exp\left(-\frac{(q - 2a^2/\sigma_n^2)}{8\,a^2/\sigma_n^2}\right) \qquad : (9)$$

Now to transformation the PDF of γ_{oc} which is in equation (6) into channel gain 'a' is given by

$$p(a) = \frac{(1+M\bar{\gamma}_1)\, 2a^{2(M-1)} \exp\left(-\frac{a^2}{\sigma_n^2}\right)}{(M-1)!\, \sigma_n^{2M}\left(1+\frac{\bar{\gamma}_1}{\sigma_n^2}a^2\right)} \quad : (10)$$

The unconditional PDF of channel LLR is calculated by averaging the equation (9) over the channel gain 'a'

$$p_0(q) = \int_0^{\infty} p(q|a, s(t) = +1)\, p(a) da \quad : (11)$$

Putting the values of equations (9&10) in equation (11), then the unconditioned PDF of channel LLR is calculated as

$$p_0(q) = \frac{(1+M\bar{\gamma}_1)}{\sqrt{2\pi}(M-1)!\,\sigma_n^{2M-1}}\left[(-1)^{M-1}\frac{\partial^{M-1}}{\partial b^{M-1}} - \frac{\bar{\gamma}_1}{\sigma_n^2}(-1)^{M}\frac{\partial^{M}}{\partial b^{M}}\right]\left[\frac{1}{2}\sqrt{\frac{\pi}{2}}\exp\left(-\sigma_n|q|\sqrt{\frac{b}{2}}+\frac{q}{2}\right)\right] \quad : (12)$$

where $b = \frac{3}{2\sigma_n^2}$ and The PDF of check node message is given by [11]

$$p_c(q) = \frac{1}{\sqrt{4\pi m_c}}\exp\left(-\frac{(q-m_c)^2}{4m_c}\right) \quad : (13)$$

where m_c is the mean of check node 'c' of degree 'i'. At (l+1) th iteration mean of check node 'c' of degree 'i' is updated by

$$m_c^{l+1} = 4\left[\mathrm{erfc}^{-1}\left(\sum_{i=0}^{d_c}\rho_i\left(1-\left(1-2P_b^l\right)\right)^{i-1}\right)\right]^2$$

The PDF of variable node is calculated by convolving the PDF of channel LLR with the PDF of check node that is

$$p_v(q) = \int_{-\infty}^{\infty} p_c(q-\tau)\, p_0(q) d\tau \quad : (14)$$

Putting the values of equations (12 & 13) in equation (14) by using the integral in [10, equation 3.322(2)], then PDF of
variable node is

$$p_v(q) = \frac{(1+M\bar{\gamma}_1)}{4\sqrt{2}(M-1)!\sigma_n^{2M-1}}\left[(-1)^{M-1}\frac{\partial^{M-1}}{\partial b^{M-1}} - \frac{\bar{\gamma}_1}{\sigma_n^2}(-1)^{M}\frac{\partial^{M}}{\partial b^{M}}\right]\left[\sqrt{\frac{1}{b}}\exp\left(\frac{\sigma_n^2 b m_c}{2}-\frac{m_c}{4}\right)\right]\left\{\left(\exp(\sigma_n\sqrt{\frac{b}{2}}+\frac{1}{2})q\right)\left[1-\right.\right.$$

Φq2mc+σnbmc2+exp(−σnb2+12)q1−Φ−q2mc+σnbmc2 :15

To obtain the probability of bit error P_e, integrate the PDF of variable node given by equation (15) from -∞ to 0,

$$P_e = \frac{(1+M\bar{\gamma}_1)}{4\sqrt{2}(M-1)!\sigma_n^{2M-1}}\left[(-1)^{M-1}\frac{\partial^{M-1}}{\partial b^{M-1}} - \frac{\bar{\gamma}_1}{\sigma_n^2}(-1)^{M}\frac{\partial^{M}}{\partial b^{M}}\right]\left[\frac{4}{\sqrt{b}-2\sigma_n^2 b^{\frac{3}{2}}}\mathrm{erfc}\left(\sigma_n\sqrt{\frac{bm_c}{2}}\right)\exp\left(\frac{\sigma_n^2 b m_c}{2}-\frac{m_c}{4}\right)+\right.$$

8σn22σn2b−1erfc(mc2) :(16)

Now at (l +1) th iteration, averaging above equation over all the bit node degrees j becomes

$$P_e = \frac{(1+M\bar{\gamma}_1)}{4\sqrt{2}(M-1)!\sigma_n^{2M-1}}\sum_{j=2}^{d_v}\lambda_k\left[(-1)^{M-1}\frac{\partial^{M-1}}{\partial b^{M-1}} - \right.$$

γ1σn2−1M∂M∂bM4b−2σn2b32erfcσnbmcl+1(j−1)2expσn2b(j−1)mcl+12−(j−1)mcl+14+8σn22σn2b−1erfc((j−1)mcl+12) :(17)

Because at (l+1) th iteration mean of variable node v of degree j is $m_v^{l+1} = (j-1)m_c^{l+1}$.

V. Result And Discussion

In this section, the analytical results for the LDPC-OC system have been presented. To evaluate the performance, the system has been considered when the number of receive antennas 'M' is taken as 3, 4,5and 6 and number of interferer is taken as 1.

For the uncoded OC system, shown in Fig.2 it is clearly observed that as numbers of receiving antennas is increased, significant diversity gain is achieved. For BER of 10^{-2}, an improvement of 1.1dB is achieved in SNR when number of receive antennas are increased from 5 to 6.

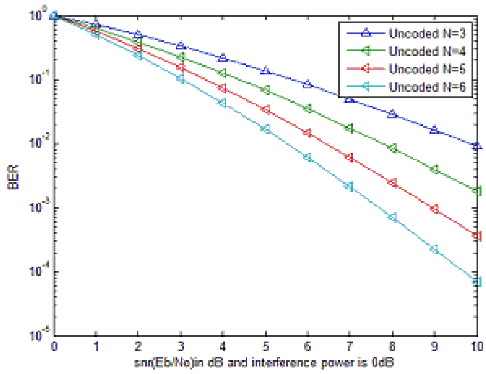

Fig.2 ber for uncoded oc system

Fig.3 presents the result of LDPC-OC system obtained by using equation (17). For the LDPC-OC system, at the BER of 10^{-4}, an improvement of 1.3 dB is achieved in SNR when number of receive antennas are raised from 5 to 6.

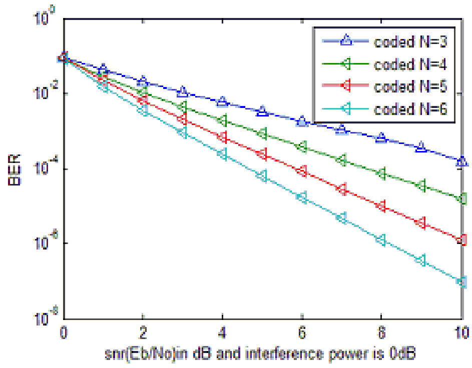

Fig.3 ber for ldpc-oc system

Comparison of uncoded OC and LDPC-OC system is given in Fig.4. It is clear from the figure that significant coding gain is achieved when comparing both the systems. At BER of 10^{-2}, a coding gain of 4.1 dB is achieved by LDPC-OC system over OC system alone when the number of receive antenna is 6.

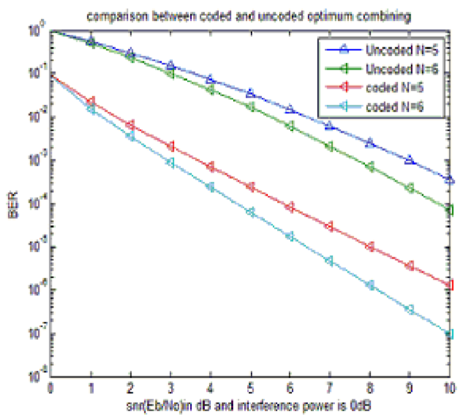

Fig.4 comparison between ber of oc and ldpc-oc system

VI. Conclusion

Using the Gaussian approximation approach, the BER expression for uncoded OC and LDPC-OC over an i.i.d rayleigh fading channel has been derived the presence of one interferers. From the numerical results, it is shown that LDPC-optimum combiner gives the more optimistic results as compared to uncoded-OC system.

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